



# MATH PRIMER

By Applied Environmental Consulting, Inc.

(Updated January 2024)

---

This math primer is designed to take a student through basic mathematical concepts that will be used in working with radioactivity concepts. Each individual on the job may not necessarily use these concepts, but they will be used throughout the 40-hour Radiation Safety Officer Training Course. This primer was developed for those that learned this math in school long ago and have forgotten most of it. After a refresher, there will be new concepts to learn that build off the math. You are welcome to download this file to your desktop or simply print out for future reference.

This primer will explain the math that is used for:

- Performing Activity conversions
- Calculating half-life and decay calculations
- Calculating changes due to time, distance and shielding
- Converting meter readings to radioactivity and exposure rates
- Calculating the dose for personnel using the Radiation Work Permit

This manual will be divided into two parts. The first part will be the **Basic Math**. It will start off with the Basics and move to the more advanced. Feel free to pass the subjects that you already know. The second part will be the **Radiation Math** which puts into practice the Math basics and applies them to Radiation Safety.

## **INDEX**

### **PART I MATH**

#### **BASIC MATH 4**

SUBJECT 1: Addition / Subtraction / Multiplication / Division .....	4
Addition, subtraction, multiplication and division are not difficult if there is only one operation.....	4
SUBJECT 2: Exponents.....	6

#### **INTERMEDIATE MATH ..... 12**

SUBJECT 3: Advanced Conversions .....	12
SUBJECT 4: Multiply or Divide by 1 .....	19
SUBJECT 5: Word problems .....	21
SUBJECT 6: Logarithms/Exponentials .....	23

### **PART II RADIATION MATH**

SUBJECT 7: Radioactivity.....	25
SUBJECT 8: Half-Life .....	29
SUBJECT 9: Half-Life Using a Calculator .....	36
SUBJECT 10: Time .....	43
SUBJECT 11: Distance (Inverse Square Law).....	44
SUBJECT 12: Shielding.....	45
SUBJECT 13: Radiation Work Permit – Exposure Limits .....	48
SUBJECT 14: Radiation Work Permit – Dose Report .....	49

## **RULES**

RULE: Do multiplication and division, then addition and subtraction.....	4
RULE: Do operations in parenthesis, then multiplication and division, and finally addition and subtraction.....	5
RULE: The number of places we move the decimal over to the left is the exponent we put by the 10. ....	6
RULE: Whatever we do to one side we do to the other.....	12
RULE: Not only the numbers, but also the letters give us important information.....	13
RULE: Anything divided by itself equals one.....	14
RULE: Anything multiplied by one remains unchanged.....	15

## PART I – MATH

### BASIC MATH

#### SUBJECT 1: Addition / Subtraction / Multiplication / Division

This section starts with the basics of math. The most basic concepts of math are addition and subtraction. During grade school, our parents started us with counting, the teachers taught us addition and subtraction. If that was not bad enough, they hit us with multiplication and division. This is where we begin.

Addition, subtraction, multiplication and division are not difficult if there is only one operation.

Addition:  $2 + 3 = 5$

Subtraction:  $9 - 5 = 4$

Multiplication:  $6 \times 2 = 12$

Division:  $9 / 3 = 3$

But what happens when more than one operation is present in an equation.

$$2 + 3 \times 12 + 6 =$$

**RULE: DO MULTIPLICATION AND DIVISION, THEN ADDITION AND SUBTRACTION.**

Generally, equations are solved left to right with the multiplication and division done first. The addition and subtraction is done next. So the equation above would be solved by first multiplying  $3 \times 12$  and then adding the 2 and the 6.

$$2 + 36 + 6 = 44$$

Sometimes, we may have a complex equation where we would want to do the addition before the multiplication. For these situations, we can use parenthesis to separate parts of the equation and indicate what needs to be done first. Each of these equations uses the same numbers in the same order. However, each has a different answer depending on where the parentheses are located.

$$(2 + 3) \times 12 + 6 = \quad (5) \times 12 + 6 = \quad 66$$

$$2 + (3 \times 12) + 6 = \quad 2 + (36) + 6 = \quad 44$$

$$(2 + 3) \times (12 + 6) = \quad (5) \times (18) = \quad 90$$

**RULE: DO OPERATIONS IN PARENTHESIS, THEN MULTIPLICATION AND DIVISION, AND FINALLY ADDITION AND SUBTRACTION.**

Here are some practice problems.

$5 \times 6 + 17 =$	47
$4 + 3 \times 2 + 9 =$	19
$4 + 3 \times (2 + 9) =$	37
$7 + 6 + 3 + 2 \times 2 =$	20
$7 + 6 + (3 + 2) \times 2 =$	23
$9 + 1 \times 5 =$	14

Some values can become very large when multiplied together.

$$1,200 \times 1,000 = 1,200,000$$

$$15 \times 6,000 = 90,000$$

Taking these numbers and multiplying them again by another number can make them too large to use easily. To handle large numbers, we want to use exponentials and scientific notation.

#### **Ratios – Write as a fraction and then convert to decimal**

1.	16 over 32 =	0.5
2.	3 over 4 =	0.75
3.	14 over 42 =	0.33
4.	42 over 14 =	3
5.	160 over 32 =	5

## SUBJECT 2: Exponents

For large numbers, with lots of zeros, scientific notation is very useful. The number of zeros in the value can be removed and the number of zeros indicated separately. For instance,

$$10,000 = 10 \times 10 \times 10 \times 10$$

There are 4 values of 10 in this equation. It would be nice if we could simply mention the number of 10s we're using. We can; its call exponentials where "4" is the exponent. In this case, the base unit of 10 is raised to the "fourth power" and the exponent is 4. The scientific notation is telling us the 10 is multiplied by itself 4 times.

$$10,000 = 10^4$$

So now let us take a more complex number such as 3,650,000,000. This value can be separated into two values multiplied together.

$$3,650,000,000 = 365 \times 10,000,000$$

$$3,650,000,000 = 365 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$$

$$3,650,000,000 = 365 \times 10^7$$

What if we have a large number that doesn't have many zeros? Can we still use this same technique? Yes. Using decimals, the previous value can be simplified until we have only one number to the left of the decimal point.

$$3,650,000,000 = 36.5 \times 100,000,000$$

$$3,650,000,000 = 36.5 \times 10^8$$

But also,

$$3,650,000,000 = 3.65 \times 1,000,000,000$$

$$3,650,000,000 = 3.65 \times 10^9$$

**RULE: THE NUMBER OF PLACES WE MOVE THE DECIMAL OVER TO THE LEFT IS THE EXPONENT WE PUT BY THE 10.**

This format is commonly used in Scientific Notation. It consists of one number to the left of the decimal point and one or more numbers to the right. All the remaining numbers are insignificant and can be removed. The value to the right of the multiplier indicates the power of the number or how many positions to move the decimal point to get the original number.

This gives us an easier way to look at exponentials - by moving the decimal point. Let's take the value of 3,650,000,000. Every time we move the decimal place to the left, we increment the value of the exponential.

$$\begin{aligned}
 3,650,000,000 &= 3650000000. \\
 3,650,000,000 &= 365000000.0 \times 10^1 \\
 3,650,000,000 &= 36500000.00 \times 10^2 \\
 3,650,000,000 &= 3650000.000 \times 10^3 \\
 3,650,000,000 &= 365000.0000 \times 10^4 \\
 3,650,000,000 &= 36500.00000 \times 10^5 \\
 3,650,000,000 &= 3650.000000 \times 10^6 \\
 3,650,000,000 &= 365.0000000 \times 10^7 \\
 3,650,000,000 &= 36.50000000 \times 10^8 \\
 3,650,000,000 &= 3.650000000 \times 10^9
 \end{aligned}$$

Now we can remove the "trailing" zeros. The value can again be written:

$$3.65 \times 10^9$$

Problems:

$5,400,000 = 5.4 \times 10^?$	6
$67 = 0.67 \times 10^?$	2
$32,000 = 32 \times 10^?$	3
$5,760 = \underline{\hspace{2cm}} \times 10^3$	5.76
$6,000,000 = \underline{\hspace{2cm}} \times 10^5$	60

If  $10^7$  means move the decimal 7 spaces to the right, what do you suppose  $10^{-7}$  means? If you guessed that the decimal is to be moved 7 spaces to the left, you are correct. Look at the following step-by-step progression.

$$\begin{aligned}
 &5.4 \times 10^{-7} \\
 &= 0.54 \times 10^{-6} \\
 &= 0.054 \times 10^{-5} \\
 &= 0.0054 \times 10^{-4} \\
 &= 0.00054 \times 10^{-3} \\
 &= 0.000054 \times 10^{-2} \\
 &= 0.0000054 \times 10^{-1} \\
 &= 0.00000054
 \end{aligned}$$

Problems:

$0.000055 = 5.5 \times 10^?$	-5
$0.0067 = 0.67 \times 10^?$	-2
$0.0000032 = 32 \times 10^?$	-7
$0.000576 = \underline{\hspace{2cm}} \times 10^{-3}$	0.576
$0.06 = \underline{\hspace{2cm}} \times 10^{-5}$	6000

Since numbers can be compacted using exponentials, the next step is to use them with math. Exponentials are best used for multiplication and division only. Let's look at a simple example.

$$100 \times 600 = 60,000$$

Using scientific notation,

$$\begin{aligned}
 &(10 \times 10) \times (6 \times 10 \times 10) \\
 &= 10^2 \times 6 \times 10^2 \\
 &= 6 \times 10^2 \times 10^2
 \end{aligned}$$

Now comes the tricky part. For multiplication using exponential values, the exponents are “added”.

$$= 6 \times 10^{(2+2)}$$

$$= 6 \times 10^4$$

$$= 60,000$$

Adding the exponential works for all multiplications using exponentials. For example:

$$(3.2 \times 10^8) \times (16 \times 10^5)$$

Since everything is being multiplied, we can rearrange values to simplify solving the equation.

$$= 3.2 \times 16 \times 10^8 \times 10^5$$

$$= 51.2 \times 10^{(8+5)}$$

$$= 51.2 \times 10^{13}$$

Or, putting it into scientific notation:

$$= 5.12 \times 10^{14}$$

Try these problems:

$$200 \times 300 = \underline{\hspace{2cm}} \qquad 6 \times 10^4$$

$$2 \times 10^4 \times 8 \times 10^2 = \underline{\hspace{2cm}} \qquad 1.6 \times 10^7$$

$$4 \times 10^3 \times 600 = \underline{\hspace{2cm}} \qquad 2.4 \times 10^6$$

$$3 \times 10^2 \times 4000 = \underline{\hspace{2cm}} \qquad 1.2 \times 10^6$$

If we were able to add the exponents when we multiplied numbers together, what is done for dividing exponentials? In this case, the exponential values are subtracted.



$$\begin{aligned} & \frac{320,000,000}{1,600,000} \\ &= \frac{32 \times 10^7}{16 \times 10^5} \\ &= \frac{32}{16} \times \frac{10^7}{10^5} \\ &= \frac{32}{16} \times 10^{(7-5)} \\ &= 2 \times 10^2 = 200 \end{aligned}$$

Problems:

$$\begin{aligned} \frac{8000}{40} &= 2 \times 10^2 \\ \frac{4 \times 10^{12}}{2000} &= 2 \times 10^9 \\ \frac{16,000,000}{200} &= 8 \times 10^4 \end{aligned}$$

Both multiplication and division can be found in the same equation. For this type of problem involving only multiplication and division, it is not important which is done first.

$$\begin{aligned} & 2 \times 10^3 \times \frac{14 \times 10^6}{7 \times 10^4} \\ &= \frac{2 \times 14 \times 10^{(3+6)}}{7 \times 10^4} \\ &= \frac{2 \times 14}{7} \times 10^{(3+6-4)} \\ &= 4 \times 10^5 \end{aligned}$$

Problems:

$$\frac{6000 \times 200}{40} = 3 \times 10^4$$

$$\frac{2000 \times 6000}{400} = 3 \times 10^4$$

**Place the following numbers in decimal format** \_\_\_\_\_

- |    |                                                  |                |
|----|--------------------------------------------------|----------------|
| 1. | $7.3 \times 10^8 =$                              | 730,000,000    |
| 2. | $3.7 \text{ E}+10 =$                             | 37,000,000,000 |
| 3. | $3.7 \times 10^8$ divided by $5.2 \times 10^7 =$ | 7.1            |

**Place the following numbers in scientific notation** \_\_\_\_\_

- |    |                                            |                      |
|----|--------------------------------------------|----------------------|
| 1. | 0.0000053 =                                | $5.3 \times 10^{-6}$ |
| 2. | 100,000,000 =                              | $1 \times 10^8$      |
| 3. | 130000097 (Put to 2 significant figures) = | $1.3 \times 10^8$    |

**Convert Decimals to Percent and Percent to Decimals** \_\_\_\_\_

- |     |          |       |
|-----|----------|-------|
| 1.  | 0.35 =   | 35%   |
| 2.  | 47.3% =  | 0.473 |
| 3.  | 0.075 =  | 7.5%  |
| 4.  | 22.5 % = | 0.225 |
| 5.  | 1.12 =   | 112%  |
| 6.  | 2% =     | 0.02  |
| 7.  | 0.0024 = | 0.24% |
| 8.  | 125% =   | 1.25  |
| 9.  | 0.29 =   | 29%   |
| 10. | 26%=     | 0.26  |

**Decimals** \_\_\_\_\_

- |    |                   |       |
|----|-------------------|-------|
| 1. | $15.1 - 7.3 =$    | 7.8   |
| 2. | $23.45 - 12.22 =$ | 11.23 |

- |    |                 |       |
|----|-----------------|-------|
| 3. | $98.2 - 17 =$   | 81.2  |
| 4. | $23 - 12.1 =$   | 10.9  |
| 5. | $41.7 - 3.35 =$ | 38.35 |

## INTERMEDIATE MATH

### SUBJECT 3: Advanced Conversions

You can multiply anything by anything. But to keep an equation the same, or equivalent, it is important to do the same thing to both sides of the equation. Let's take a simple example from everyday life.

1 Apple costs 25 cents

In equation form,

$$1 \text{ Apple} = \$0.25$$

How much would 5 Apples cost?

Because this type of math is so common to us, we can easily do this in our head and immediately know the answer to be \$1.25. But how do we show this on paper?

**RULE: WHATEVER WE DO TO ONE SIDE WE DO TO THE OTHER.**

We first multiply one side by 5. In order to keep the equation the same, we also have to multiply the other side by 5.

$$1 \text{ Apple} \times 5 = \$0.25 \times 5$$

$$5 \text{ Apples} = \$1.25$$

This matches what we did in our head. Not only can we do this with multiplication, we can also do this with division. Here's an example.

You can buy 6 limes for one dollar

In equation form,

$$6 \text{ limes} = \$1.00$$

What if we wanted only 3 limes? What does it cost?

Again, we can do this in our head and come up with an answer of \$0.50. But how do we show buying half the quantity of limes?

$$\frac{6 \text{ limes}}{2} = \frac{\$1.00}{2}$$

$$3 \text{ limes} = \$0.50$$

Now let's look at some notations. We'll use the last example where 6 limes cost 1 dollar.

$$6 \text{ limes} = \$1.00$$

We know that 1 dollar is the same as 4 quarters.

$$\$1.00 = 4 \text{ Quarters}$$

Replacing that with the first equation gives us:

$$6 \text{ limes} = 4 \text{ Quarters}$$

This can become cumbersome, so let's just use some abbreviations. Let's use "L" for limes and "Q" for quarters.

$$6 \text{ L} = 4 \text{ Q}$$

As long as we know what the letters represent, we can use anything we want. A lime is just a green, sour ball and a quarter is a shiny, round disc. So we can use those terms and we are still describing Limes and Quarters.

$$6 \text{ green, sour balls} = 4 \text{ shiny, round discs}$$

We could take it one step farther and just use the initials.

$$6 \text{ gsb} = 4 \text{ srd}$$

We know that “gsb” is a *green, sour ball*, aka a Lime and “srd” is a *shiny, round disk* aka a quarter. Since a quarter is \$0.25, we also know that an “srd” is equal to \$0.25. In equation form,

$$1 \text{ srd} = \$0.25$$

**RULE: NOT ONLY THE NUMBERS, BUT ALSO THE LETTERS GIVE US IMPORTANT INFORMATION.**

We use a value of radiation exposure called the milli-Roentgen, you will learn all about that later. We abbreviate it as “mR.” Many of our survey meters give us results in mR per hour (mR/hr). When performing a survey, results are written in mR/hr. Here is a typical survey.

Source #1, Left Side at 1 meter:	3.2 mR/hr
Source #1, Right Side at 1 meter:	2.5 mR/hr
Source #1, Top at 1 meter:	2.3 mR/hr
Source #1, Bottom at 1 meter:	1.5 mR/hr

Some survey meters provide their results in counts per minute or “cpm.” (another topic you will learn later). In practice, you will see the abbreviation in upper case or lower case (cpm). On the side of the survey meter is usually a conversion of cpm to mR/hr. For example:

$$2600 \text{ cpm} = 1 \text{ mR/hr}$$

When working with mathematics, it’s not important to know what the letters mean, just that they are important. Let’s see how to manipulate them.

**REMEMBER: Whatever I do to one side, I must do to the other.**

Let’s assume we want results in **mR/hr**. What is a **mR/hr**? If you don’t know, don’t worry about it. This is only for the sake of math. Let’s assume we have a piece of equipment that reads in units called **cpm**. What is a **cpm**? Who knows and who cares just yet. But let’s say we have a conversion and know how many **cpms** there are in a **mR/hr**.

Let’s assume that  $2600 \text{ cpm} = 1 \text{ mR/hr}$ .

Again, don’t worry about the letters – it’s just a “thing”.

Using the equation above for conversion of **cpm** to **mR/hr**, we can divide both sides of the above equation by the same “thing” to keep the equation equivalent. In this case, we divide both sides by 2600 cpm.

$$\frac{2600 \text{ cpm}}{2600 \text{ cpm}} = \frac{1 \text{ mR/hr}}{2600 \text{ cpm}}$$

**RULE: ANYTHING DIVIDED BY ITSELF EQUALS ONE.**

The left side of the equation falls under the rule where something divided by itself equals 1.

$$1 = \frac{1 \text{ mR/hr}}{2600 \text{ cpm}}$$

Now if we encounter a reading in cpm, we can convert to mR/hr. How?

If we have a piece of equipment that gives us a reading of 52,000 cpm, what would the reading be in mR/hr?

Basic math has shown we can multiply any number by 1 and it doesn't change the value.

$$52,000 \text{ cpm} \times 1 = 52,000 \text{ cpm}$$

$$52,000 \text{ cpm} \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 52,000 \text{ cpm}$$

**RULE: ANYTHING MULTIPLIED BY ONE REMAINS UNCHANGED.**

We can substitute the conversion above for the number 1.

$$\begin{aligned} 52,000 \text{ cpm} &= 52,000 \text{ cpm} \\ &= 52,000 \text{ cpm} \times \frac{1 \text{ mR/hr}}{2600 \text{ cpm}} \end{aligned}$$

The cpm divided by cpm equals 1

Maybe another way of displaying this equation will help you to understand that cpm is actually divided by cpm

$$\begin{aligned}
 &= \frac{52,000 \text{ cpm}}{1} \times \frac{1 \text{ mR/hr}}{2600 \text{ cpm}} \\
 &= \frac{52,000 \text{ cpm} \times 1 \text{ mR/hr}}{2600 \text{ cpm}} \\
 &= 52,000 \times \frac{1 \text{ mR/hr}}{2600} \\
 &= 20 \text{ mR/hr}
 \end{aligned}$$

Lets look at some other equations routinely used when working with radiation. (don't worry about dpm or dps just yet. For intro to algebra, dpm = \_\_ and dps = \_\_.

$$60 \text{ dpm} = 1 \text{ dps}$$

Divide both sides by 1 dps.

$$\begin{aligned}
 \frac{60 \text{ dpm}}{1 \text{ dps}} &= \frac{1 \text{ dps}}{1 \text{ dps}} \\
 \frac{60 \text{ dpm}}{1 \text{ dps}} &= 1
 \end{aligned}$$

Using the same type of set up as before, how many dpm are there if we have 360 dps?

$$360 \text{ dps} \times \frac{60 \text{ dpm}}{1 \text{ dps}} = 21,600 \text{ dpm}$$

Another manipulation of equations is to cross and multiply. For example, if we have a fraction equaling another fraction and the unknown is in the denominator (bottom part), we can cross and multiply to remove the fraction. Note the following example:

When converting from cpm to dpm in a survey, we need to know the efficiency of the instrument, that is, how well is it actually detecting what we want to measure. The survey meter can only detect a fraction of the total number of emissions from a source during a survey. The efficiency of a meter is simply the number of counts that are detected by the meter divided by the number of emissions from a source. This is determined by a calibration facility. The equation for that is:

$$\frac{\text{cpm}}{\text{dpm}} = \text{efficiency}$$

So an efficiency of 40% means that the survey meter “counts” 40 out of every 100 disintegrations per minute. So we can say that our survey meter has an efficiency of 40 “counts” per minute (cpm) per 100 disintegrations per minute (dpm).

$$\frac{40 \text{ cpm}}{100 \text{ dpm}} = \text{efficiency}$$

If we know the cpm (from the survey meter) and the efficiency, and our dpm is the unknown. An unknown can be designated as any symbol. For this example, the unknown will be “Y”. In order to solve this, we must get the “Y” all by itself and in the numerator (top part). So, if our cpm is 10,000 and our efficiency is 0.4 (meaning 40% or 40 cpm per 100 dpm), our equation sets up to be:

$$\frac{10,000 \text{ cpm}}{Y \text{ dpm}} = \frac{0.40 \text{ cpm}}{\text{dpm}}$$

Note: the percentage is changed to a fraction, that is, from 40% to 0.40. 40% is the same as 0.40 of 1. Also, 0.40 (cpm/dpm) can be considered as the numerator with 1 as the denominator.

We “cross and multiply” by multiplying the bottom part of one side with the top part of the other. (Note that 10,000 cpm multiplied by 1 is 10,000 cpm)

$$10,000 \text{ cpm} = \left( \frac{0.40 \text{ cpm}}{\text{dpm}} \right) \times Y \text{ dpm}$$

Then to solve for Y, we divide both sides by 0.40:(cpm/dpm)

$$\frac{10,000 \text{ cpm}}{0.40 \text{ (cpm/dpm)}} = \frac{0.40 \text{ (cpm/dpm)} \times Y \text{ dpm}}{0.40 \text{ (cpm/dpm)}}$$

Each side then is reduced to,



$$25,000 \text{ dpm} = Y \text{ dpm}$$

25,000 dpm is the answer. For the question “What is the activity, in dpm, for a survey meter reading 10,000 cpm with a survey meter efficiency of 40%.”

Problems:

Survey meter has 30% efficiency

$$100 \text{ cpm} = \underline{\hspace{2cm}} \text{ dpm} \qquad 333 \text{ dpm}$$

$$250 \text{ cpm} = \underline{\hspace{2cm}} \text{ dpm} \qquad 833 \text{ dpm}$$

Survey meter has 10% efficiency

$$100 \text{ cpm} = \underline{\hspace{2cm}} \text{ dpm} \qquad 1000 \text{ dpm}$$

$$250 \text{ cpm} = \underline{\hspace{2cm}} \text{ dpm} \qquad 2500 \text{ dpm}$$

Survey meter has 3% efficiency

$$100 \text{ cpm} = \underline{\hspace{2cm}} \text{ dpm} \qquad 3333 \text{ dpm}$$

$$250 \text{ cpm} = \underline{\hspace{2cm}} \text{ dpm} \qquad 8333 \text{ dpm}$$

**Solve for X**

$$1. \quad 2X + 6 = 18 \qquad X = \qquad 6$$

(Hint: remember your rules. Start by subtracting 6 from each side of the equation, which preserves the equality. Next, divide each side of the equation by 2 to solve for X.)

$$2. \quad 2X + 6 = 3X + 4 \qquad X = \qquad 2$$

$$3. \quad 16 - 12X = 4 \qquad X = \qquad 1$$

$$4. \quad 19 - 6X = 4X - 11 \qquad X = \qquad 3$$

$$5. \quad 54 - 3X = 6X - 9 \qquad X = \qquad 7$$

#### SUBJECT 4: Multiply or Divide by 1

We already touched on a rule in math that is always a constant. You can multiply anything by 1 and it doesn't change the value.

$$4 \times 1 = 4$$

$$5 \times 1 = 5$$

$$5 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 5$$

This is also true for non-numeric values.

$$\text{Apple} \times 1 = \text{Apple}$$

$$\text{Bag of cement} \times 1 = \text{Bag of cement}$$

$$\text{Cigarette} \times 1 = \text{Cigarette}$$

This value of "1" can take many forms. There is no reason it has to be just the numeric "1". As long as it has a "value" of 1, we can use it in multiplication.

Let's take a value of 220 dps and multiply it by 1.

$$220 \text{ dps} \times 1 = 220 \text{ dps}$$

Let's use a value of 1 that was defined earlier.

$$\frac{60 \text{ dpm}}{1 \text{ dps}} = 1$$

Take the value of 1 in the first equation and replace it with the second equation.

$$220 \text{ dps} \times \frac{60 \text{ dpm}}{1 \text{ dps}} = 220 \text{ dps}$$

Then we can solve the equation. We can move values around in the equation to make it easier for us come up with an answer.

$$\frac{220 \text{ dps}}{1} \times \frac{60 \text{ dpm}}{1 \text{ dps}} = 220 \text{ dps}$$

$$\frac{220 \text{ dps}}{1} \times \frac{60 \text{ dpm}}{1 \text{ dps}} = 220 \text{ dps}$$

$$\frac{220 \text{ dps}}{1 \text{ dps}} \times \frac{60 \text{ dpm}}{1} = 220 \text{ dps}$$

From a previous lesson, we know that anything divided by itself equals 1. Therefore, dps/dps = 1.

$$\frac{220}{1} \times \frac{60 \text{ dpm}}{1} = 220 \text{ dps}$$

From here we can finish the math to come up with an answer.

$$220 \times 60 \text{ dpm} = 220 \text{ dps}$$

$$13,200 \text{ dpm} = 220 \text{ dps}$$

We have just learned that 220 dps equals 13,200 dpm.

We have taken a value equivalent to “1” and used it to convert a number in one unit (dps) to different unit (dpm).

Let’s try this with another unit conversion. Take a survey meter that is reading 1000 cpm. We want to convert that to mR/hr. We can use a previous value of 1 to help.

We already know from the label on our survey meter that:

$$1 \text{ mR/hr} = 2600 \text{ cpm}$$

$$1 = \frac{1 \text{ mR/hr}}{2,600 \text{ cpm}}$$

So how many mR/hr would you find if the survey meter reading is 1000 cpm?

$$1000 \text{ cpm} \times \frac{1 \text{ mR/hr}}{2,600 \text{ cpm}} =$$

Again we can manipulate and move around values that are only multiplied and divided.

$$\frac{1000 \text{ cpm}}{1} \times \frac{1 \text{ mR/hr}}{2,600 \text{ cpm}} =$$

$$\frac{1000 \text{ cpm}}{1} \times \frac{1 \text{ mR/hr}}{2,600 \text{ cpm}} =$$

$$\frac{1 \text{ mR/hr}}{1} \times \frac{1,000 \text{ cpm}}{2,600 \text{ cpm}} =$$

Anything divided by itself is equal to 1.

$$\frac{1 \text{ mR/hr}}{1} \times \frac{1,000}{2,600} =$$

$$\frac{1 \text{ mR/hr}}{1} \times \frac{1,000}{2,600} =$$

From here we can solve the equation.

$$\frac{1 \text{ mR/hr}}{1} \times \frac{1,000}{2,600} = 0.38 \text{ mR/hr}$$

Try these problems using the same conversion factor above. (Conversion = 1mR/hr / 2600 cpm)

$$1,600 \text{ cpm} = \underline{\hspace{2cm}} \text{ mR/hr} \qquad 0.62$$

$$20,000 \text{ cpm} = \underline{\hspace{2cm}} \text{ mR/hr} \qquad 7.7$$

### SUBJECT 5: Word problems

Word problems are used everyday by everyone. If a six-pack of beer costs \$5.00, then how much would two six-packs cost? \$10.00

$$\$5.00 \times 2 = \$10.00$$

Wow, gas prices are \$5.00 per gallon. How much is it going to cost to fill my 20-gallon tank?

$$\$5.00/\text{gallon} \times 20 \text{ gallon} = \$100.00$$

If the next city is 50 miles away and we can only drive 45 miles per hour, roughly how long will it take to arrive? Intuitively we know that it's going to take over an hour.

$$\frac{50 \text{ miles}}{45 \text{ miles/hr}} = 1.1 \text{ hr}$$

Now let's see how this works for us with radiation. If someone stands in a 2 mR/hr radiation field for 3 hours, what will be their total exposure?

$$\begin{aligned} 2 \text{ mR/hr} \times 3 \text{ hr} &= \frac{2 \text{ mR}}{\text{hr}} \times 3 \text{ hr} \\ &= \frac{2 \text{ mR}}{\text{hr}} \times \frac{3 \text{ hr}}{1} \\ &= \frac{3 \text{ hr}}{\text{hr}} \times \frac{2 \text{ mR}}{1} \\ &= \frac{3}{1} \times \frac{2 \text{ mR}}{1} \\ &= 3 \times 2 \text{ mR} \\ &= 6 \text{ mR} \end{aligned}$$

Try these problems.

What if someone were in a 5 mR/hr field for 3 hours. What would be their exposure?

15 mR

What if someone were in a 2 mR/hr field for a 12 hour shift. What would be their exposure?

24 mR

Inverse Square Law:

If the exposure rate at 1 foot is 6 mR/hr, at what distance would be you need to be to get to 2 mR/hr?

If the exposure rate at 1 foot is 15 mR/hr, what is the exposure rate at 10 feet?

### SUBJECT 6: Logarithms/Exponentials

From Lesson 2, very large or very small numbers were made more manageable through the use of exponentials. Listed below are some examples.

Number	Exponential Expression
1000	$10^3$
100	$10^2$
10	$10^1$
1	$10^0$
0.1	$10^{-1}$
0.01	$10^{-2}$
0.001	$10^{-3}$

Logarithms can simplify these numbers even more by removing the need for the base unit of 10. So for the list above, the logarithms would be as follows.

Number	Exponential Expression	Logarithms
1000	$10^3$	3
100	$10^2$	2
10	$10^1$	1
1	$10^0$	0
0.1	$10^{-1}$	-1
0.01	$10^{-2}$	-2
0.001	$10^{-3}$	-3

Logarithms are not only good for whole number multiples of 10, but for all numbers. What would be the logarithm of 1.1?

From the list above,  $1 = 10^0$  and  $10 = 10^1$ . So the value of 1.1 would have an exponent between 0 and 1. Using a calculator, the log of 1.1 is 0.0414. We can write this out as:

$$1.1 = 10^{0.0414}$$

The log of 1.1 would be,

$$\log(1.1) = \log(10^{0.0414}) = 0.0414$$

Here are some other values to try in the calculator. To learn more about calculators, go to Section 8.

$$\log(2.1) = 0.32$$

$$\log(14) = 1.15$$

$$\log(230) = 2.36$$

$$\log(7470) = 3.87$$

$$\log(0.17) = -0.77$$

$$\log(0.004) = -2.40$$

Multiplication and division of logarithms is accomplished by adding or subtracting the exponents.

$$10^x \times 10^y = 10^{x+y}$$

$$10^x / 10^y = 10^{x-y}$$

Use a calculator and solve these:

$\log(7) \times \log(20) =$	1.099
$\log(0.7) \times \log(20) =$	-0.202
$150 \times \log(15) =$	176.4
$10 \times \log(10) =$	10

If it's possible to use other than whole numbers as the exponents, then is it possible to use a number other than 10 as the base? Yes!

---

## PART II – RADIATION MATH

The following sections supplement the Radiation Safety Training program. We call it Radiation Math because it is math you will come in contact with when dealing with radioactivity and worker safety.

### SUBJECT 7: Radioactivity

#### Conversions

Multiply number of:	By	To obtain number of:
becquerel (Bq)	$2.703 \times 10^{-11}$	curies (Ci)
becquerel (Bq)	27.03	picocuries (pCi)
curies (Ci)	$3.700 \times 10^{10}$	dis/sec (dps)
curies (Ci)	$2.220 \times 10^{12}$	dis/min (dpm)
curies (Ci)	$10^3$	millicuries (mCi)
curies (Ci)	$10^6$	microcuries (uCi)
curies (Ci)	$10^9$	nanocuries (nCi)
curies (Ci)	$10^{12}$	picocuries (pCi)
curies (Ci)	$10^{-3}$	kilocuries (kCi)



curies (Ci)	$3.700 \times 10^{10}$	becquerel (Bq)
nanocuries (nCi)	37	becquerel (Bq)
dis/min (dpm)	$4.505 \times 10^{10}$	millicuries (mCi)
dis/min (dpm)	$4.505 \times 10^{-7}$	microcuries (uCi)
dis/sec (dps)	$2.703 \times 10^{-8}$	millicuries (mCi)
dis/sec (dps)	$2.703 \times 10^{-5}$	microcuries (uCi)
gray (Gy)	100	rad
kilocuries (kCi)	$10^3$	curies (Ci)
microcuries (uCi)	$3.700 \times 10^4$	dis/sec (dps)
microcuries (uCi)	$2.220 \times 10^6$	dis/min (dpm)
millicuries (mCi)	$3.700 \times 10^7$	dis/sec (dps)
millicuries (mCi)	$2.220 \times 10^9$	dis/min (dpm)
rad	$10^{-2}$	gray (Gy)

**REMEMBER:**  $3.7 \times 10^{10}$  dps = 1 Ci

$$1 \text{ dps} = 1 \text{ Bq}$$

$$1 \text{ millicurie (mCi)} = 10^{-3} \text{ Ci}$$

$$1 \text{ microcurie } (\mu\text{Ci}) = 10^{-6} \text{ Ci}$$

$$1 \text{ nanocurie (nCi)} = 10^{-9} \text{ Ci}$$

$$1 \text{ picocurie (pCi)} = 10^{-12} \text{ Ci}$$

Example:

$$3.7 \times 10^{10} \text{ dps} = \underline{\hspace{2cm}} \text{ curies (Ci)}$$

Let's use the Conversion table above to convert from **dps** to **Ci**.

Multiply number of:	By	To obtain number of:
dis/sec (dps)	$2.703 \times 10^{-11}$	curies (Ci)

$$3.7 \times 10^{10} \times 2.703 \times 10^{-11} =$$

Rearrange the numbers

$$3.7 \times 2.703 \times 10^{10} \times 10^{-11} =$$

$$10 \times 10^{(10-11)} =$$

$$10 \times 10^{-1} = 1$$

1.  $3.7 \times 10^7 \text{ dps} = \underline{\hspace{2cm}}$  curies

$$1 \times 10^{-3}$$

2.  $1000 \text{ mCi} = \underline{\hspace{2cm}}$  curies

Convert the numbers and prefixes to powers of 10:

$1000 = (10^3)$  and  $m = (10^{-3})$ ; now, we rewrite the equation to read:

$(10^3) (10^{-3}) \text{ Ci} = \underline{\hspace{1cm}} \text{ Ci}$ . Now combine the exponents...  $(10^{+3})(10^{-3}) = 10^0$ ;

Since  $10^0$  is equal to 1. Our answer is 1 Ci.

3.  $100 \text{ mCi} = \underline{\hspace{2cm}}$  microcuries

There are several ways to do these conversions. One can go to a conversion table and do the simple math. Or put the numbers in like units, such as powers of 10. For example in this problem:

$$100 \text{ mCi} = \underline{\hspace{1cm}} \text{ microcuries}$$

Since  $100 = 10^2$  and a millicurie =  $(10^{-3})$ .... and  $(10^{-6})$  equals a micro, we have

$$1 \times 10^2 \times 10^{-3} \text{ curies} = \underline{\hspace{1cm}} (10^{-6}) \text{ curies.}$$

We add our exponents since we are multiplying...

$$\text{This equals } 1 \times 10^{-1} \text{ curies} = \underline{\hspace{1cm}} (10^{-6}) \text{ curies.}$$

What power of 10 (exponent) would balance that equation?  $10^5$

So,  $1 \times 10^{-1}$  curies =  $1 \times (10^5) (10^{-6})$  curies; Thus, 100 mCi =  $10^5$  microcuries

4.  $1 \text{ Ci} = \underline{\hspace{2cm}}$  microcuries

$1 \times 10^6$

5.  $2.2 \times 10^9 \text{ dpm} = \underline{\hspace{2cm}}$  mCi

$2.2 \times 10^9 \text{ dpm} = \underline{\hspace{2cm}}$  mCi

To begin, we need to know how many dpm are in a curie.

$3.7 \times 10^{10} \text{ d/s} = 1 \text{ Ci}$ .

To get from disintegration per second to disintegrations per minute...

Multiply (60 seconds/1 minute) times... $3.7 \times 10^{10} \text{ dis/sec} (60 \text{ sec/min}) = 2.22 \times 10^{12} \text{ dis/min}$

Divide our number by one ( $2.22 \times 10^{12} \text{ dpm/1Ci}$ ).

(Dividing our number of:  $2.2 \times 10^9 \text{ dpm} / (2.22 \times 10^{12} \text{ dpm/1Ci}) = .99 \times 10^{-3} \text{ Ci}$   
 (The 1 Ci in the denominator of a denominator, puts it in the numerator. Thus, our unit is the Ci.

Since  $(10^{-3})$  = a millicurie. Our answer is **0.99 mCi**.

6.  $0.005 \text{ uCi} = \underline{\hspace{2cm}}$  nCi

5

7.  $2.2 \times 10^6 \text{ dpm} \underline{\hspace{2cm}}$  microcuries

0.99

8. We have 100 mCi of Co-60, 20 mCi of Ra-226 and 15 mCi of Cs-137. What is our total radioactivity? *Since the problem doesn't ask for the total radioactivity for any specific radionuclide, we add them all together.*

135 i

9. We have  $3.7 \times 10^7$  dps of C-14 and  $3.7 \times 10^8$  dps of Mn-54. What is our total radioactivity?

*In this case, we are ADDING the radioactive materials; however, they have different powers of 10. We need to make the powers of ten the same in order to simply add the integers. We do this by converting one of the numbers to be the same power of ten as the other. For example,*

*$3.7 \times 10^7$  dps of C-14 plus  $37 \times 10^7$  dps of Mn-54 is the same as above.*

*The Mn-54 value of  $3.7 \times 10^8$  and  $37 \times 10^7$  are the same.*

*$3.7 \times 10^7$  dps (C-14) +  $37.0 \times 10^7$  dps (Mn-54)*

*$40.7 \times 10^7$  or  $4.07 \times 10^8$  dps (for the correct scientific notation format)*

The total activity of the material?  $4.07 \times 10^8$  dps

10. 1 Bq equals \_\_\_\_\_ pCi? 27

11. 1 pCi equals \_\_\_\_\_ Bq? 0.037

## SUBJECT 8: Half-Life

For calculations involving radioactive materials, our first concept is the characteristic called **half-life**. This is the time for half of the amount of a certain radioactive material to decay. If we know the “original” activity “ $A_0$ ” of a source of radiation, then we can calculate the amount of radioactive material left “ $A_t$ ” after a period of time (t). We use logarithms with a base of one-half (0.5). The following is the equation for calculating decay.

$$A_t = A_0 \times e^{-\lambda t}$$

Or expressed more concisely and without the multiplication sign:

$$A_t = A_0 e^{-\lambda t}$$

Where:

$A_t$  = the activity at time t

$A_0$  = the original activity

$e$  = the number used as a base in calculus (has a value of 2.718281828). In our application, the number following the “e” in the numerator is an exponent that needs to be calculated with the  $e^x$  function on your calculator.

$\lambda$  = lambda is the decay constant. It’s the natural logarithm of 2 (which has a value of (-0.69314718) divided by the half-life of the radionuclide in the source

t = the elapsed time since the source activity was determined (note: this is any time lapse of interest to you)

We solve this equation for an unknown  $A_t$  in four parts.

Part 1: Calculate  $\lambda$ . The natural logarithm (“ln” on your calculator) of 0.5 (for half-life) has a value of -0.69314718, so you can enter the rounded number of -0.693 and divide it by the half-life of the radionuclide.

$$-\lambda = -0.693 / t_{1/2}$$

Part 2: Calculate the exponent. Multiply the “ $\lambda$ ” by the time “t”. This is the time interval from the original measurement to sometime in the future. For example, if you wanted to know the activity of some source 290 days into the future, your “t” would be 290 days.

$$\lambda t = 0.693t / t_{1/2}$$

Part 3: Calculate the base  $e$  result. With the number for the exponent displayed on your calculator, hit **SHIFT** and the  **$e^x$**  key for the answer to this part. It will be a positive number, and always less than one.

Part 4: Calculate  $A_t$ . This result is multiplied by the original activity of the radionuclide " $A_0$ " to determine the activity at some future time " $A_t$ " (note:  $A_t$  will have the same units of activity as  $A_0$ , e.g. Bq, Ci, mCi, etc.)

Putting it all together.

For example, we have 100 mCi of iodine-125 today. Iodine-125 has a half-life of 60 days. What would the activity be in 60 days? Intuitively, we know that it would be one-half, or 50 mCi. But let's do the math.

$$\begin{aligned} A_0 &= 100 \text{ mCi} \\ t &= 60 \text{ days} \\ t_{1/2} &= 60 \text{ days} \end{aligned}$$

$$A_t = (100 \text{ mCi}) \times e^{-((0.693/60 \text{ days}) \times 60 \text{ days})}$$

Which is the same as:

$$A_t = (100 \text{ mCi}) \times e^{-(0.693) / (60 \text{ days}) \times 60 \text{ days}}$$

Note: "days" are in the same time units and will cancel out because one of them is in the numerator and the other is in the denominator

$$A_t = (100 \text{ mCi}) \times e^{-(0.01155 \times 60)}$$

$$A_t = (100 \text{ mCi}) \times e^{-0.693}$$

(now we use the  $e^x$  function for the -0.693 which = 0.5)

So, we multiply the result times our original activity = (100 mCi) x 0.5

$$A_t = 50 \text{ mCi}$$

What would the activity have been in only 30 days?

Put the 30 days in instead of 60 days for the “t”.

It changes the e number to be  $-0.693 \times .5 = -0.3465$ .

Use the  $e^x$  function with a result of .707

Multiply the result times the original activity of 100 mCi = 70.7 mCi

## Problems

Pd-103 has a 17-day half-life.

What would 350 mCi become in 8 days?	252 mCi
What would 350 mCi become in 28 days?	112 mCi
What would 350 mCi become in 1 year?	$1.2 \times 10^{-4}$ mCi

Cs-137 has a 30-year half-life.

What would 0.5 mCi become in 10 years?	0.4 mCi
What would 0.5 mCi become in 35 years?	0.2 mCi

What if we didn't want to know the amount of activity in the future given an activity now? What if we needed to know the activity now given an activity in the future?

$$A_t = A_0 e^{-\lambda t}$$

$$A_t = A_0 e^{-0.693 t/t_{1/2}}$$

We need the initial activity ( $A_0$ ) by itself.

**Remember the rule: whatever you do to one side you must do to the other.**

$$\frac{A_t}{e^{-0.693 t/t_{1/2}}} = A_0$$

Example:

We want 360 uCi I-125 in 3 weeks (21 days). I-125  $t_{1/2} = 60$  days. What do we need to pull out of our inventory today?

$$\frac{360 \text{ uCi}}{e^{-0.693 \times (21 \text{ days} / 60 \text{ days})}} = A_0$$

$$A_0 = \frac{360 \text{ uCi}}{e^{-0.24255}}$$

$$A_0 = \frac{360 \text{ uCi}}{0.785}$$

$$A_0 = 459 \text{ uCi}$$

What if we have products with an activity of 700 uCi and our client wants only 260 uCi products. How long do we need to wait?

$$A_t = A_0 e^{-\lambda t}$$

$$A_t = A_0 e^{-0.693 t / t_{1/2}}$$

Divide both sides by  $A_0$

$$\frac{A_t}{A_0} = e^{-0.693 t / t_{1/2}}$$

To remove the "e", we need to take the natural log (ln) of both sides of the equation.

$$\ln\left(\frac{A_t}{A_0}\right) = \ln\left(e^{-0.693 t / t_{1/2}}\right)$$

$$\ln\left(\frac{A_t}{A_0}\right) = -0.693 t / t_{1/2}$$

$$\ln\left(\frac{A_t}{A_0}\right) (-0.693) t_{1/2} = t$$

$$\ln\left(\frac{260 \text{ uCi}}{700 \text{ uCi}}\right) X (-0.693) X 60 \text{ days} = t$$

$$t = 84.3 \text{ days}$$



Does this answer make sense? Yes. We know that in one half-life the activity will reduce from 700 uCi to 350 uCi. We still need a lower number, so the product will need to decay past at least one half-life. At two half-lives, the activity will reduce from 700 uCi to 175 uCi. This is too low. So we know our answer is somewhere between one and two half-lives. 84.3 day fits nicely between one half-life (60 days) and two half-lives (120 days).

### Half-Life

1. How much activity will remain if 300 mCi of Cobalt 60 ( $T_{1/2} = 5$  years) is stored for 10 years?

*(Another way of thinking of this type of problem is if the number of years elapsed is a multiple of the half-life, one can do it in their head, can't they. For example, 300 mCi after one half-life (5 years) would be 150 mCi. After another half-life (total of 10 years), it's half again, or 75 mCi.)*

75 mCi

2. If we have 25 mCi of Tc99m ( $T_{1/2} = 6$  hrs), what was its activity at this same time yesterday?

*In this problem, which unknown is it that we are looking for in our equation? It's the original activity ( $A_0$ ).*

*So, in setting up our equation, we would have:  $A_t = A_0 e^{-0.693 t / T_{1/2}}$*

*Plugging in our numbers:  $25 \text{ mCi} = A_0 e^{-0.693(24\text{days}/6\text{days})}$*

*The "days" cancel out to leave numbers only.*

*Now we can calculate: Doing the exponent first,  $e^{-2.772}$  Then, using the  $e^x$  function, we get 0.0625*

*So,  $25 \text{ mCi} = A_0 (.0625)$  Now, we isolate the unknown ( $A_0$ ) by dividing both sides by 0.0625*

*$25 \text{ mCi}/0.0625 = A_0$  Now do simple division which  $A_0$  equals 400 mCi.*

3. We have a nuclear density gauge using Cs-137 having 250 mCi at the time of purchase. If the source is no longer useful at the 125 mCi level, when do we need to replace the source? ( $T_{1/2} = 30$  yrs.)

30 yr

4. If we have isotope A ( $T_{1/2}$  of 6 mo.) and isotope B ( $T_{1/2}$  of 30 yrs), having the same activity now; assuming all else equal, which isotope had more activity a year ago? Which isotope is a more significant waste disposal problem?

*This is not a mathematical problem, simply a common-sense problem. The shorter the half-life, results in more radioactivity earlier, and the longer the half-life results in more activity later.*

*So, what do you think are the answers?*

*A had more activity a year ago*

*B is the more significant waste problem*

5. Using the decay equation from Lesson 6, calculate what percentage of Cs-137 ( $T_{1/2} = 30$  yr) activity remains after being onsite for 10 years. In other words, what percentage of its activity remains today compared to when you received it? Could this affect the performance of the gauge/detector system if the cut-off for effective use is that 70% of the original activity must remain?

79% remains, NO

12. Using the decay formula, we have Cs-137, ( $T_{1/2} = 30$  yr) from a previous owner. When purchased it was 100 mCi. They used it 7 years and you intend to use it for 5 years. If you need to maintain at least a 70% of the original activity level, is it worth purchasing?

YES, 76% still remains after 12 years

13. There is an unknown radioactive material with a half-life of 34 hours. The total radioactivity at 11:00 a.m. today was 678 mCi. What will the activity be at noon tomorrow?

407 mCi

14. The radioactivity of our Cesium-137 source today is 68 mCi. When did we acquire the source if the original activity was 125 mCi?

*In this problem, we have to determine what is our unknown. Our  $A_t$  is 68 mCi. Our  $A_o$  is 125 mCi. Our half-life is 30 years (since we are dealing with Cesium-137). So, the remaining unknown is the "t" (time).*

*So, in plugging the numbers we have:  $68 \text{ mCi} = 125 \text{ mCi} e^{-.693 t/30}$*

*Let's clean this up a little and isolate our unknown...*

*We divide both sides by 125 mCi and do as much math in the exponent that we can.... $68 \text{ mCi}/125 \text{ mCi} = e^{-0.0231 (t)}$*

*The mCi cancel out resulting in  $.544 = e^{-0.0231 t}$*

*In order to get the unknown into a number (instead of an exponent), we take the natural log of both sides, which removes the exponent.*

$$\ln(0.544) = \ln(e^{-0.0231 t}) = -.0231 t$$

$$-0.609 = -.0231 t$$

*To isolate "t", we divide both sides by -.0231 which cancels the -.0231 next to the "t". Our result so far is:  $-.609/-.0231 = t$*

*The minus signs cancel out to yield 26.36 years ago*

## **SUBJECT 9: Half-Life Using a Calculator**

Calculators have been available for 50 years. The simplest calculator back then was cumbersome. Today they are included in wristwatches and smart phones. Inexpensive basic scientific calculators can do much more than what is required for this course. Every equation provided in this primer can be solved using a calculator with basic logarithmic functions.

Many common calculators use solar cells powered by light. Even fluorescent lighting in buildings is sufficient to provide energy. No batteries are needed and the calculator will remain on as long as the solar strip – usually located at the top of the calculator – is exposed to light. To turn it off, place the cover back over the calculator. Please refer to this Casio fx-260 solar calculator as an example.



The calculator buttons have dual uses. In normal use, the symbols on the buttons themselves are active. For this course, a few of those buttons will not be used. So, effectively, the calculator could look like this:




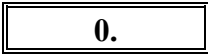

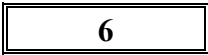

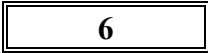

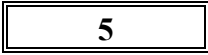

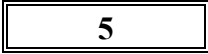
However, things are not always simple. There are a few other functions that we will need on the calculator. The orange-colored symbols above the buttons are accessed only when the SHIFT key is pressed. After the SHIFT key is pressed, only the next button pressed will use the orange functions. The following button press reverts back to the representation on the button itself. Fortunately, only a few SHIFT functions are needed:



As a first introduction to using the calculator, let's do a simple equation.

$$6 + 5 \times 2$$

The following is a list of the buttons to press and the display that should appear. Before entering any equation, always clear the calculator by pressing the AC button. Pressing it more than once will not hurt the calculator. To complete the entry, the "=" key must be pressed.

Key	Display
	
	
	
	
	

2	2
=	16

The calculator has automatically performed the equation using math hierarchy. Lesson 1 of this primer describes math hierarchy. But briefly, multiplication and division are done first and then addition and subtraction. But what if it was our desire to do the addition first and then the multiplication? In that event, we would use parenthesis. Operations in parenthesis are always performed before multiplication and division.

So let's try this equation using parenthesis:

$$(6 + 5) \times 2$$

Key	Display
AC	0.
(	01 0.
6	6
+	6
5	5
)	11
X	11
2	2
=	22

Now we will move rapidly ahead to equations from Lesson 7. The first one is calculating decay.

In words, we are looking for the decay of a radionuclide with a 22-day half-life that has an activity of 0.35 mCi today and we wish to know what the activity will be in 17 days. We will enter the following equation:

$$A_t = A_0 e^{-\lambda t}$$

$$A_t = A_0 e^{-0.693 t/t_{1/2}}$$

$$A_0 = 0.35 \text{ mCi}$$

$$t = 17 \text{ days}$$

$$t_{1/2} = 22 \text{ days}$$

The equation will be

$$A_t = 0.35 \times e^{-(0.693 \times 17 \text{ days} / 22 \text{ days})}$$

In this equation, we will need to use the function  $e^x$  immediately after calculating the negative exponent.

**NOTE:** The calculator uses the  $\div$  sign while we will use the  $/$ . They are the same and mean that we are performing division.

We will first determine the exponential portion of the equation, 17 days / 22 days, and place it into memory: Placing values into memory requires use of the SHIFT key and the Min key.

Key	Display
AC	0.
.693	0.693
X	0.693
17	17



÷	11.781
22	22
=	0.5355
+/-	-0.5355
SHIFT	-0.5355
$e^x$	0.585376528
x	0.585376528
.35	0.35
=	0.204881785

Use this to try solving the following problems. The answers are given in the right column.

I-125 has a half-life of 60 days. For 30 mCi of I-125 on 21 April 2020, what would be the activity on 21 April 2021?

0.44 mCi

A sample of Rn-222 with an activity of 150 pCi on 12 June is to be shipped on 25 June of the same year. What is going to be the activity? Rn-222 has a half-life of 3.823 days.

14.2 pCi

Th-234 has a half-life of 24.1 days. What is the activity of a 5 mCi sample 30 days later?

2.1 mCi

Starting Activity	Half-Life	Time Interval	Answer
0.35 mCi	60 days	45 days	0.21 mCi
25 mCi	30 years	15 years	17.68 mCi
25 mCi	30 years	30 years	12.5 mCi
25 mCi	30 years	45 years	8.84 mCi
100 mCi	30 minutes	1 hour	25.01 mCi
1000 Ci	30 minutes	1 days	3.58E-12 Ci
47 $\mu$ Ci	17 days	6 days	36.8 $\mu$ Ci

### SUBJECT 10: Time

Time is important for radiation safety. Think of a fire. You can brush your hand through the fire without damage. But hold your hand there and damage will occur.

As a radiation worker, you are allowed 5,000 mrem of dose per year.

For gamma rays, we know that an exposure rate of 1 mR/hr is equivalent to a dose rate of 1 mrem/hr.

If I am in a 5 mR/hr field for 5 hours, what is my dose at the end of those 5 hours?

$$5 \text{ mR/hr} = 5 \text{ mrem/hr}$$

$$5 \text{ mrem/hr} \times 5 \text{ hr} = 25 \text{ mrem}$$

What is my dose if I work in a 2 mR/hr field for 45 minutes?

$$2 \text{ mR/hr} = 2 \text{ mrem/hr}$$

$$2 \text{ mrem/hr} \times 0.75 \text{ hr} = 1.5 \text{ mrem}$$

Problems:

6 mR/hr for 10 minutes

1 mrem

4 mR/hr for 2 hours

8 mrem

10 mR/hr for 30 minutes

5 mrem

## SUBJECT 11: Distance (Inverse Square Law)

The calculator can also be used to calculator Inverse Square Law problems. This law states that radiation intensity changes by the square of the distance under certain conditions. For instance, moving from 1 meter from a source to 2 meters from the source doubles the distance but decreases the exposure rate to one fourth.

This equation works for X-radiation and gamma-radiation when the source of radiation can be approximated as being a small point and distances to exposure points are relatively large. If so, the exposure rate at measurement location 1 multiplied by the distance to that location squared is equal to the exposure rate at a more distant location 2 multiplied by the distance to that second location squared.

$$\text{Exposure Rate}_1 \times \text{Distance}_1^2 = \text{Exposure Rate}_2 \times \text{Distance}_2^2$$

So, if the Exposure Rate<sub>1</sub> is 1 mR/hr at 1 meter from the point source, and the Exposure Rate<sub>2</sub> is unknown but is 2 meters from the source:

$$1 \text{ mR/hr} \times (1\text{meter})^2 = \text{Exposure Rate}_2 \times 2\text{m}^2$$

$$1 = \text{Exposure Rate}_2 \text{ in mR/hr} \times 4$$

$$\text{Exposure Rate}_2 = 1/4 = 0.25 \text{ mR/hr}$$

Let's calculate the exposure rate at a distance of 15 feet when the exposure rate is 0.5 mR/hr at 10 feet.

$$\text{Exposure Rate}_1 = 0.5 \text{ mR/hr}$$

$$\text{Distance}_1 = 10 \text{ feet}$$

$$\text{Distance}_2 = 15 \text{ feet}$$

Exposure Rate<sub>2</sub> = unknown

$$\text{Exposure Rate}_1 \times \text{Distance}_1^2 = \text{Exposure Rate}_2 \times \text{Distance}_2^2$$

Exposure Rate<sub>2</sub> = 0.22 mR/hr

Use this to try solving the following problems. The answers are given in the right column.

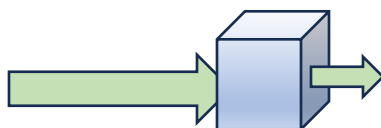
Exposure Rate	Distance	Move to	Answer
2 mR/hr	4 meters	2 meters	8.00 mR/hr
2 mR/hr	4 meters	6 meters	0.89 mR/hr
10 mR/hr	3 feet	10 feet	0.90 mR/hr
10 mR/hr	3 feet	3 yards	1.11 mR/hr
1 mR/hr	1 meter	3 meters	0.11 mR/hr
2 mR/hr	4 meters	1 meter	32.00 mR/hr
4 mR/hr	7 meters	3 meters	21.78 mR/hr
6 mR/hr	3 meters	4 meters	3.38 mR/hr
0.1 mR/hr	10 meters	1 meter	10.00 mR/hr

Note that the exposure rate always goes **up** if you move **closer** to the source and it always goes **down** as you move **away** from the source. It helps when solving for the unknown if you always assign location 1 as closest to the source and location 2 as farthest from the source. Note that a distance could also be the unknown in the problem rather than the exposure rate.

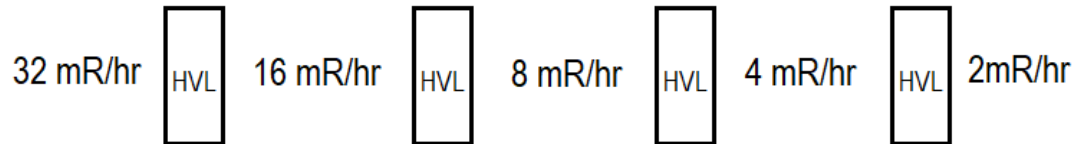
## SUBJECT 12: Shielding

### Half-Value Layers

The intensity of radiation going through a “half-value layer” of any type of material reduces the intensity on the other side of the material by one-half.



Example: We have 32 mR/hr at the source. How many Half-Value Layers (HVL) is needed to bring the exposure rate (intensity) to 2 mR/hr?



4 HVLs will reduce the exposure rate from 32 mR/h to 2 mR/hr.

The HVL for lead when shielding Cs-137 is 0.65 cm. Therefore, we need 2.6 cm of lead to reduce the exposure rate from 32 mR/hr to 2 mR/hr.

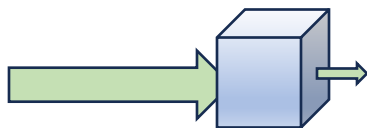
The HVL for concrete for shielding Cs-137 is 4.8 cm. Therefore, we need 19.2 cm of concrete to reduce the exposure rate from 32 mR/hr to 2 mR/hr.

How many HVLs are needed to reduce exposure rates from 9 mR/hr to 1 mR/hr?

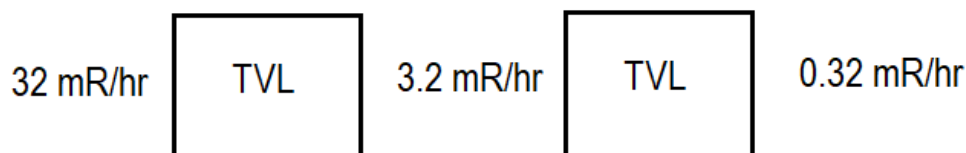
How many HVLs are needed to reduce exposure rates from 100 mR/hr to 2 mR/hr?

### Tenth-Value Layers

The intensity of radiation going through a “tenth-value layer” of any type of material reduces the intensity on the other side of the material by one-tenth.



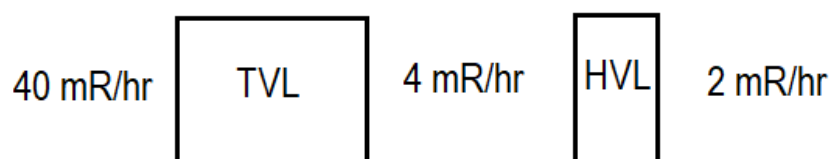
Example: Just like before, we have 32 mR/hr at the source. How many Tenth-Value Layers (TVL) is needed to bring the exposure rate (intensity) to below 2 mR/hr?



The TVL for lead when shielding Cs-137 is 2.2 cm. Therefore, we need 4.4 cm of lead to reduce the exposure rate from 32 mR/hr to below 2 mR/hr.

This would be more expensive than needed. So we can combine using HVL and TVL.

Example: This time we have 40 mR/hr at the source. What combine of Tenth-Value Layers (TVL) and Half Value Layer (HVL) is needed to bring the exposure rate (intensity) to 2 mR/hr?



What combine of Tenth-Value Layers (TVL) and Half Value Layer (HVL) is needed to bring the exposure rate (intensity) from 65 mR/hr to 2 mR/hr?

What combine of Tenth-Value Layers (TVL) and Half Value Layer (HVL) is needed to bring the exposure rate (intensity) from 150 mR/hr to 2 mR/hr?

Try this real-life scenario. You have a worker that has a shield they can work behind. The shield provides one HVL. Working behind the shield, the worker will take one hour to complete the task. Without the shield, the worker can complete the job in 15 minutes.

Do you recommend the shield or no shield.?

The source is 12 mR/hr.

With the HVL, the worker will experience an exposure rate of 6 mR/hr.

The worker will work for one hour:

$$4 \text{ mrem/hr} \times 1 \text{ hr} = 4 \text{ mrem}$$

Working without the shield, the worker is done in 15 minutes (1/4 hour)

$$12 \text{ mrem/hr} \times \frac{1}{4} \text{ hr} = 3 \text{ mrem}$$

In this case, the worker will receive less dose working without the shield.

Problem:

Source is 100 mR/hr. If your worker uses a 2 HVL shield, it will take 2 hours.

If your worker uses a 1 HVL shield, it will take 1 hour.

Without any shield, the worker is done in 30 minutes.

Which do you recommend?

All give 50 mrem

### SUBJECT 13: Radiation Work Permit – Exposure Limits

This section supports the Radiation Work Permit calculations for the maximum time a worker can remain at or near the radioactive source and still keep their dose below allowable limits. The limit for our exposure rate is 2 mR/hr.

This equation will tell us how many minutes per hour a worker can remain at or near the source.

$$\frac{60 \text{ min/hr}}{\left(\frac{\text{Maximum Field Measured at Source Surface (___) mR/hr}}{2 \text{ mR/hr}}\right)} = \text{___ min/hr}$$

Let's assume the Radiation Field Measured **4 mR/hr**.

$$\frac{60 \text{ min/hr}}{\left(\frac{\text{Maximum Field Measured at Source Surface (4) mR/hr}}{2 \text{ mR/hr}}\right)} = ?? \text{ min/hr}$$

Let's remove some of the extra language.

$$\frac{60 \text{ min/hr}}{\left(\frac{4 \text{ mR/hr}}{2 \text{ mR/hr}}\right)} = ?? \text{ min/hr}$$

Start simplifying. 4 divided by 2 becomes 2 and the "mR/hr" cancel each other out.

$$\frac{(60 \text{ min/hr})}{\left(\frac{4}{2}\right)} = ?? \text{ min/hr}$$

$$\frac{(60 \text{ min/hr})}{2} = ?? \text{ min/hr}$$

60 divided by 2 becomes 30 min/hr

Does this make sense? If you are allowed to work for a full hour in a field of 2 mR/hr, then a radiation field twice as strong (4 mR/hr) would mean you could only work half as long.

Try some of these:

Exposure rate measured	Length of Time
0.5 mR/hr	unlimited
5 mR/hr	24 min
3 mR/hr	40 min
Be careful – always make sure your units match	
500 uR/hr	unlimited

#### **SUBJECT 14: Radiation Work Permit – Dose Report**

These are more real scenarios for removal of density gauges at many industrial plants.

##### **Job #1**

The #1 digester process line density gauge needs to be moved to storage since the line is being taken out of service for extended maintenance. The density gauge is a 100 mCi Ohmart Cs-137 unit model number 7142, serial number 34567. Your gamma survey meter Ludlum Model 2401 serial number is 12345 and was calibrated last August 14.

The job will utilize 2 mechanics named William Tell and Robin Hood as well as a crane operator named Billy Bright who works for the company. The gauge unit source will be removed from the pipe, lowered to the ground by the crane and transported to the storage area.

The survey readings with the source closed are 2 mR at the surface and 1 mR at 1 ft.

Tell's time was 2 minutes in contact with the density gauge and 6 minutes at 1 ft.



Hood did not have contact with the density gauge but worked 10 minutes at 1 ft.

Bright spent no time around the source.

Advanced Authorized User spent 2 minutes in contact with the density gauge and 4 minutes at 1 ft.

**REMEMBER: The Dose Rate of a gamma ray in mrem is the same as the Exposure Rate of a gamma ray in mR. i.e. 1 mR/hr = 1 mrem/hr.**

Personnel	DOSE AT CONTACT				DOSE AT 1 FOOT			Dose (mrem)
	Fraction of hour in 6-minute increments for each task	X	Exposure rate in mR/hr at surface	+	Fraction of hour in 6-minute increments for each task	X	Exposure rate in mR/hr at 1 foot	
Tell	0.1	X	2	+	0.1	X	1	0.3
Hood	0	X	2	+	0.2	X	1	0.2
Bright	0	X	2	+	0	X	1	0
AU	0.1	X	2	+	0.1	X	1	0.3

## Job #2

The #1 slurry line density gauge needs a gasket to be replaced on the spool piece that the gauge is mounted on. The density gauge is a 500 mCi Cs-137 Texas Nuclear unit model number TN12, serial number 45678. Your gamma survey meter is a Ludlum Model 3 with serial number is 23456 calibrated last October 14. The job will utilize 2 mechanics named William Tell and John McIntoch as well as a crane operator named Gil Favor who work for the company. The work includes removal of the gauge unit from the pipe. Removal of the pipe section, installation of a new gasket and reinstallation of the gauge unit.

The survey readings with the shutter closed are 3 mR at the surface and 1 mR at 1 ft.

After installation the readings with the shutter open are 6 mR at the surface and 2 mR at 1 ft.

Tell's time was 0 minutes in contact with the density gauge and 20 minutes at 1 foot.

Mclintoch's time was 0 minutes in contact with the density gauge and 10 minutes at 1 foot.

Favor spent no time around the source.

Advanced Authorized User spent 4 minutes in contact with the density gauge and 4 minutes at 1' with the shutter closed and 4 minutes at 1 foot with the shutter open.

Personnel		DOSE AT CONTACT				DOSE AT 1 FOOT			
		Fraction of hour in 6-minute increments for each task	X	Exposure rate in mR/hr at surface	+	Fraction of hour in 6-minute increments for each task	X	Exposure rate in mR/hr at 1 foot	Dose (mrem)
Tell	Closed	0	X	3	+	0.3	X	1	0.3
Tell	Open	0	X	6	+	0	X	2	
Mclintock	Closed	0	X	3	+	0.2	X	1	0.2
Mclintock	Open	0	X	6	+	0	X	2	
Favor	Closed	0	X	3	+	0	X	1	0
Favor	Open	0	X	6	+	0	X	2	
AU	Closed	0.1	X	3	+	0.1	X	1	0.6
AU	Open	0	X	6	+	0.1	X	2	

Will Tell got 0.3 mrem dose.

John Mclintock got 0.2 mrem dose.

Gil Favor got 0 mrem dose.

The AU got 0.6 mrem dose.